

RESEARCH ARTICLE

Classical-to-Quantum Transfer of Geometric Phase for Controlled Manipulation of Quantum State

Vimlesh Kumar¹  | Chahat Kaushik^{1,2}  | M. Ebrahim-Zadeh^{3,4}  | C. M. Chandrashekar⁵  | G. K. Samanta¹ 

¹Photonic Sciences Lab, Physical Research Laboratory, Ahmedabad, Gujarat, India | ²Indian Institute of Technology-Gandhinagar, Gujarat, India | ³Institutio Catalana de Recerca i Estudis Avançats (ICREA), Barcelona, Spain | ⁴ICFO-Institut de Ciències Fòniques, Castelldefels (Barcelona), Spain | ⁵Quantum Optics & Quantum Information, Department of Electronic and Systems Engineering, Indian Institute of Science, Bengaluru, India

Correspondence: Chahat Kaushik (kaushik.28chahat@gmail.com)

Received: 27 January 2026 | **Revised:** 25 August 2026 | **Accepted:** 3 September 2026

Keywords: entropy | geometric phase | Pancharatnam–Berry phase | spontaneous parametric down conversion

ABSTRACT

The geometric phase, arising from the cyclic/non-cyclic evolution of a polarization state of light, is typically measured using interferometric methods that often introduce unwanted dynamic phase contributions. Here, we present a phase measurement technique based on the quantum correlation of pair photons to measure the geometric phase of a classical beam without any dynamic phase contribution. The transfer of the geometric phase of the classical pump beam onto polarization-entangled pair photons generated via spontaneous parametric down-conversion in a Sagnac interferometer scheme enables easy control over the quantum state. Characterization of the quantum states reveals that the geometric phase of the pump beam controls the coincidence counts, entanglement visibility, Bell's parameter, quantum-state tomography, and fidelity, in close agreement with theoretical predictions. We observe sinusoidal modulation of Bell's parameter and state fidelity with changes in the geometric phase, resulting in transitions between orthogonal Bell states and Bell-like maximally entangled states. Our results establish the geometric phase of the classical pump as a tunable parameter for quantum state control, offering a compact, passive platform for phase manipulation in quantum photonic systems, enabling geometric-phase-controlled quantum operations, and compensating for the unwanted phase acquired by the quantum state during propagation.

1 | Introduction

The geometric phase, also known as the Pancharatnam–Berry phase [1, 2], arises from the cyclic evolution of a classical or quantum state on its parameter space, has found widespread applications across classical and quantum optics, including quantitative phase imaging [3], clock interferometry for enhanced metrology [4], optimal output coupling in oscillators [5], frequency shifting [6], and quantum gates for quantum computing and quantum key distribution [7]. In optics and photonics, the geometric phase is readily implemented through the evolution of the polarization state of the light beam on the Poincaré sphere, typically by propagation through various combinations of wave

plates sequentially [8]. Such a simple realization of the dispersion-free phase has enabled access to robust, broadband elements and systems for beam-shaping [9], atomic optics [10], generation of spatially structured beams [11], and advanced interferometry and sensing [12, 13].

Despite significant advances, conventional interferometric methods for measuring the geometric phase [14, 15] remain challenging due to unavoidable contributions from the dynamic phase. Efforts have been made to mitigate this issue using stable Sagnac interferometers [5], which reduce the need for active optical path-length stabilization. More recently, non-interferometric approaches have gained attention, including the

use of anisotropic lenses composed of multiple spatial light modulators [16], polarization-dependent diffraction and polarimetry [17], and nonlinear parametric processes such as stimulated second harmonic generation [18]. Nevertheless, there remains a need for novel, phase measurement techniques that offer simpler experimental implementation and straightforward estimation of the geometric phase free from dynamic phase in both classical and quantum systems.

On the other hand, imprinting the geometric phase to quantum states through the evolution of polarization on the Poincaré sphere has gained momentum. Notable examples include the generation of energy-time entanglement [19], realization of universal quantum gates [20] by imprinting geometric phase onto pair photons generated through spontaneous parametric down-conversion (SPDC) process [21], and the observation of phase memory in NOON ($N = 2$) states by transferring either the geometric phase [22–24] or dynamic phase [25] of the pump beam. The direct transfer of amplitude and phase from the pump field to SPDC photon pairs has been extensively studied, beginning with the seminal work [26] and followed by numerous investigations [24, 25]. This phase-transfer mechanism has enabled the generation of orbital-angular-momentum states [27], hybrid entangled states [28], and, more recently, an efficient technique for characterizing quantum states in arbitrary spatial modes by mapping their amplitude and phase through SPDC [29]. Despite the well-established studies of phase transfer or geometric-phase memory effects in SPDC process, the deterministic control of phase of the maximally entangled Bell-states [30–32] using a purely geometric phase and its complete quantum-resource characterization have not been experimentally demonstrated. Such states underpin a wide range of quantum technologies, including quantum communication [33], quantum teleportation [34], and quantum information processing [7, 35].

Here, we report a simple and novel experimental scheme for the measurement of the geometric phase and control of quantum states through the phase memory effect. By transferring the geometric phase of a classical pump beam, induced by rotating a half-wave plate, to polarization-entangled pair photons generated via spontaneous parametric down-conversion (SPDC) in a Sagnac interferometer, we imprint the phase memory of the pump onto the pair photons. While the Sagnac configuration is formally an interferometer, its common-path geometry ensures intrinsic phase stability of the quantum state against any dynamical phase fluctuation and eliminates the need for active interferometric phase locking in the generation process. The measurement of the geometric phase of a classical beam through the coincidence counts, entanglement visibility, Bell's parameter, quantum state tomography, and fidelity of the quantum state confirms the absence of dynamic phase contribution in the phase measurement method. Importantly, no interference fringes or path-dependent phase measurements are employed to extract or tune the relative phase of the entangled state. Using the geometric phase of the pump as a control parameter, we demonstrate state transitions between orthogonal Bell states, typically achieved through Pauli gates, and Bell-like maximally entangled states. The current findings result in a new pathway for realizing phase-dependent quantum operations in a simple and compact experimental setup, and pave the way for future implementation in integrated photonic quantum circuits.

2 | Theory

We employ the analytical approach to understand the transfer of the geometric phase (GP) of the classical pump beam to the entangled two-photon state and establish the relation of GP to the entanglement and the Clauser–Horne–Shimony–Holt (CHSH) form of Bell's inequality [36] (S). To introduce GP into the classical pump beam, we have used the Jones matrix [37] formalism for the QHQ setup consists of a quarter-wave plate (QWP, Q), a half-wave plate (HWP, H), and another quarter-wave plate (QWP, Q), sequentially positioned with their fast axes oriented at angles 45° , θ_H , and 45° , with respect to the horizontal, respectively. The phase imprinted on the classical pump beam can also be understood intuitively as arising from the rotation of its polarization state on the Poincaré sphere induced by the sequence and orientation of the waveplates. Depending on the waveplate configuration and the rotation of their fast axes, the input polarization may undergo either cyclic or non-cyclic evolution on the Poincaré sphere. In such evolutions, the dynamical phase accumulated by the individual polarization components contributes only a constant global phase and remains independent of the half-wave plate rotation angle. By contrast, according to the definition of the geometric (Pancharatnam–Berry) phase, the relative phase acquired by the beam is determined by the solid angle subtended by the polarization trajectory for both cyclic and non-cyclic paths at the center of the Poincaré sphere. Thus, the geometric-phase interpretation highlights the topological and path-dependent nature of phase control and provides a simple, general framework for quantifying the induced phase.

The transfer matrix for the QHQ can be written as [5]

$$T = QHQ = \begin{pmatrix} e^{2i\theta_H} & 0 \\ 0 & -e^{-2i\theta_H} \end{pmatrix}. \quad (1)$$

Now, for the classical pump beam of diagonal polarization, $|D\rangle = 1/\sqrt{2}(|H\rangle + |V\rangle)$, (a vector sum of horizontal and vertical polarizations), on propagation through the QHQ setup transform into the polarization state given as $1/\sqrt{2}(|H\rangle + |V\rangle) \rightarrow 1/\sqrt{2}(e^{2i\theta_H}|H\rangle - e^{-2i\theta_H}|V\rangle)$. The $|H\rangle$ and $|V\rangle$ polarization states acquire geometric phases of equal magnitude, $\phi = 2\theta_H$, but with opposite signs, dictated by the solid angle subtended by their cyclic evolution paths on the Poincaré sphere in opposite directions. The polarization projection of the beam for diagonal/anti-diagonal and horizontal/vertical basis results in laser power of $P_{DA} = (\sin^2 2\theta_H |D\rangle + \cos^2 2\theta_H |A\rangle)$ and $P_{HV} = 1/2(|H\rangle + |V\rangle)$, respectively. Therefore, the measurement of the laser at one of the output ports of PBS will result in the power variation for diagonal/anti-diagonal basis and constant power in horizontal/vertical basis for the rotation of half-wave plate (H) of QHQ setup; thus, the geometric phase.

Now the generation of entangled photons using the pump beam of polarizations state, $1/\sqrt{2}(e^{i\phi}|H\rangle - e^{-i\phi}|V\rangle)$ from a type-0 phase-matched crystal in a the polarization-based Sagnac interferometer [38, 39], results in the entangled state given as,

$$\begin{aligned} |\psi_{\phi}^{\pm}\rangle &\rightarrow \frac{1}{\sqrt{2}}(e^{i\phi}|V\rangle_s|V\rangle_t - e^{-i\phi}|H\rangle_s|H\rangle_t) \\ &\equiv \frac{1}{\sqrt{2}}(|H\rangle_s|H\rangle_t - e^{i2\phi}|V\rangle_s|V\rangle_t). \end{aligned} \quad (2)$$

Here, the subscripts denote the signal (s) and idler (i) photons. Neglecting the global phase, we find that the output state, as given by Equation 2, resembles a Bell state, with a relative phase introduced by the geometric phase between the orthogonal components of the pump beam. If the state $|\psi_{\phi}^{-}\rangle$ propagates through a birefringent medium, then the $|H\rangle_s|H\rangle_i$ and $|V\rangle_s|V\rangle_i$ will acquire different phases, let's say ϕ_H and ϕ_V , respectively, and alter the entangled state. In such a case, the resultant state can be

$$|\psi_{\phi}^{-}\rangle \equiv \frac{1}{\sqrt{2}}(|H\rangle_s|H\rangle_i - e^{i(2\phi+\phi_V-\phi_H)}|V\rangle_s|V\rangle_i). \quad (3)$$

It is evident from Equation (3) that by suitably adjusting the geometric phase, ϕ , one can make the $2\phi + \phi_V - \phi_H = 0$ to compensate dispersion of the birefringent medium and restore the desired quantum state [24, 40].

To study the generated state as shown by Equation (2), we calculate, using the simple mathematical treatment [41, 42], the coincidence probability after polarization projection for the signal and the idler photon as

$$P(\Theta_s, \Theta_i) = \frac{1}{2}(\cos^2 \Theta_s \cos^2 \Theta_i + \sin^2 \Theta_s \sin^2 \Theta_i - \frac{1}{2} \sin 2\Theta_s \sin 2\Theta_i \cos 2\phi). \quad (4)$$

Here, Θ_s and Θ_i denote the inclination angles of the polarizers with respect to the x-axis, placed in the signal and idler arms, respectively. Using the Equation (4), the probability of projecting the state onto the H , D , V , and A bases can be determined by substituting values of Θ_s as 0° , 45° , 90° , 135° and scanning Θ_i from 0° to 180° . Again, it is evident from Equation (4), that the coincidence probability of the state is dependent on the geometric phase ϕ . For the D/A basis resulting from the combination of (Θ_s, Θ_i) values of $(\pm 45^\circ, \pm 45^\circ)$, the coincidence probability is given as

$$P = \frac{1}{4}(1 - \cos 2\phi). \quad (5)$$

From Equation (5), we can see that the coincidence probability of the generated state in the D/A measurement basis depends on the geometric phase. On the other hand, in the H/V basis, the coincidence probability ($P = 1/2$) remains independent of the geometric phase of the input classical pump beam. Therefore, one can measure the coincidence of the pair photons to measure the geometric phase.

To verify the quantum nature, using Equation (4), we calculated the CHSH form of Bell's inequality [36] of the generated quantum state as,

$$\begin{aligned} C(\Theta_s, \Theta_i) &= P(+, \Theta_i, +, \Theta_s) + P(-, \Theta_i, -, \Theta_s) \\ &\quad - P(+, \Theta_i, -, \Theta_s) - P(-, \Theta_i, +, \Theta_s) \\ &= [(\cos 2\Theta_s \cos 2\Theta_i) - \sin 2\Theta_s \sin 2\Theta_i \cos 2\phi]. \end{aligned} \quad (6)$$

Here, $C(\Theta_s, \Theta_i)$, the correlation between the two observables identified by the polarizer angles, Θ_s and Θ_i , is averaged over the ensemble of all possible outcomes. Also, the outcome $+/-$ is

identified as the emergence and no emergence of photons from the polarizer, respectively. Using Equation (6), we find the CHSH Bell's parameter, S , associated with the angles $(\Theta_s, \Theta_i, \Theta'_s, \Theta'_i) = (0^\circ, 22.5^\circ, 45^\circ, \text{ and } 67.5^\circ)$, as

$$\begin{aligned} S &= |C(\Theta_s, \Theta_i) - C(\Theta_s, \Theta'_i)| + |C(\Theta'_s, \Theta_i) + C(\Theta'_s, \Theta'_i)| \\ &= |\sqrt{2}| + |\sqrt{2} \cos 2\phi|. \end{aligned} \quad (7)$$

We can see that CHSH Bell's parameter, S , of the output pair photons state (given by Equation 2) depends on the geometric phase of the parent classical pump beam [43, 44]. It is also interesting to note that the CHSH Bell's parameter has a maximum value of $2\sqrt{2}$ for GP of $\phi = 0^\circ$ and a minimum of $\sqrt{2}$ for $\phi = 45^\circ$. On the other hand, using the von Neumann entropy of the reduced density matrix approach [45, 46], we calculated the entanglement entropy of subsystem A in the bipartite system $A \cup B$, given by $S_A = -\text{Tr}(\rho_A \log \rho_A)$, where ρ_A is obtained by tracing out subsystem B . The two-qubit state represented by Equation (2) results in an entanglement entropy of unity, independent of the geometric phase, confirming that the quantum state is maximally entangled. On the other hand, the observed decrease in the CHSH Bell's value with the geometric phase indicates a deviation of the generated quantum state from the standard Bell state, even though the entanglement entropy remains unchanged. To achieve maximal violation of the CHSH Bell's inequality for arbitrary values of ϕ , it is necessary to determine the optimal measurement bases. Thus, the combined variation of the CHSH Bell's value and entanglement entropy captures the quantum nature of the geometric phase.

Further, we can calculate the fidelity of the geometric phase induced quantum state, $|\psi_{\phi}\rangle$, with the initial state, $|\psi_{\phi=0}^{-}\rangle$, as

$$F = |\langle \psi_{\phi} | \psi_{\phi=0}^{-} \rangle|^2 = \cos^2 \phi. \quad (8)$$

The Equation (8) reveals that the quantum state fidelity depends on the geometric phase of the classical pump beam. As the fidelity $F = 0$ for $\phi = 90^\circ$, we observe the generation of orthogonal states for $\phi = 0$ and 90° . Thus, by tuning the geometric phase of the classical pump beam, we can make the transition between two orthogonal Bell states. Furthermore, by measuring the fidelity of the output state, we can also determine the geometric phase. This derivation highlights the possibility of transferring geometric phase from a classical system to a nonclassical state and subsequently measuring it through quantum parameters using any of the equations, Equation (5), (7), and (8), without using any interferometer commonly used for geometric phase measurement. Moreover, the geometric phase of the pump beam, which can be manipulated by simply rotating a single wave plate of QHQ combination, can be adjusted to compensate for the relative phase acquired by the entangled state during propagation through different media.

3 | Experimental Methods

The schematic of the experimental setup is shown in Figure 1. A single-frequency, linearly polarized (vertically polarized) continuous-wave (cw) laser (Toptica, TopMode) [27, 38], providing 40 mW of single-frequency output at 405 nm, is used

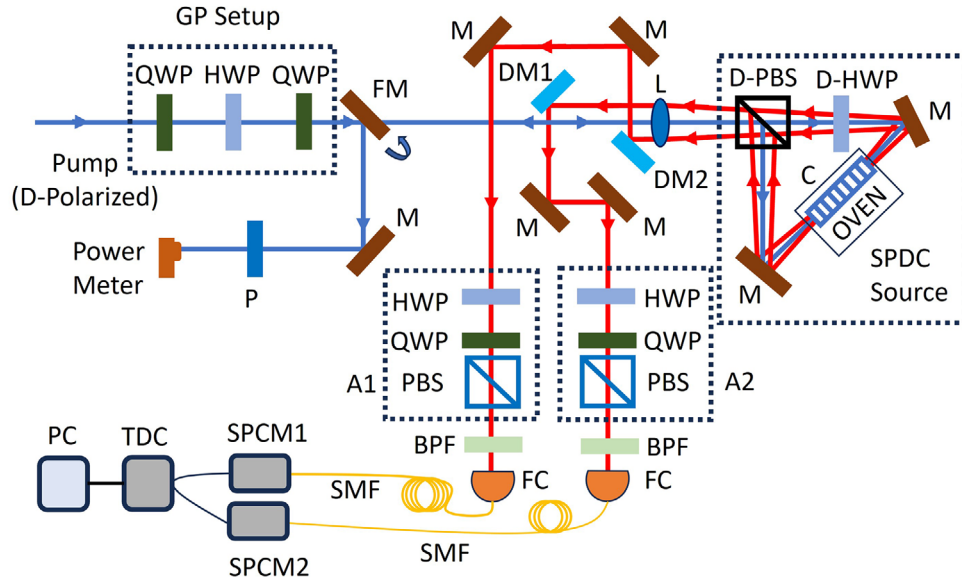


FIGURE 1 | Schematic experimental setup for studying the classical geometric phase using quantum states. Blue line denotes the pump at 405 nm, and red line denotes SPDC photons at 810 nm; QWP, Q: quarter wave plate; HWP, H: half wave-plate; GP: geometric phase; FM: flip mirror; M: mirror; P: polarizer, combination of HWP and polarizing beams splitter at 405 nm; D-PBS: dual-wavelength (405 and 810 nm) polarizing beam-splitter; D-HWP: dual-wavelength (405 and 810 nm) half-wave plate; C: PPKTP crystal; DM1-2: D-shaped mirror; A1-2: polarization analyzers consist of HWP, QWP, and PBS; FC: Fiber collimator; SMF: single mode fibre; BPF: bandpass filter; SPCM: single photon counting module; TDC: time-to-digital converter.

as the pump laser in the experiment. A half-wave plate (not shown in Figure 1) transforms the laser output into diagonal (D) polarization. The pump beam then propagates through the geometric phase setup (GP setup), which consists of a quarter-wave plate (QWP, Q), a half-wave plate (HWP, H), and another quarter-wave plate (QWP, Q), arranged in a QHQ configuration with their optic axes oriented at 45° , θ_H , and 45° with respect to the horizontal, respectively. As a result, the horizontal and vertical components of the pump beam acquire geometric phases of the same magnitude but with opposite signs.

The geometric phase is imprinted on the pump beam by simply rotating the HWP of the GP setup and verified by using a flip mirror (FM) on the beam path to direct the pump beam for polarization projection using the polarizer (P), a combination of HWP and polarization beam splitter (PBS) optimized for 405 nm, and the power meter. A lens (L) of focal length $f = 150$ mm is used to focus the pump at the center of a 30-mm long and 2×1 aperture, periodically poled PPKTP crystal (C) of a single grating period of $3.425 \mu\text{m}$. The crystal is mounted on the temperature oven to maintain the crystal temperature of 27°C with a stability of $\pm 0.1^\circ \text{C}$ for type-0 ($e \rightarrow e + e$), non-collinear phase-matched, degenerate SPDC photons at 810 nm. The crystal, C, is placed inside the polarization Sagnac interferometer, comprised of a dual-wavelength (405 and 810 nm) polarization beam splitter cube (D-PBS), dual-wavelength half-wave plate (D-HWP), and mirrors M, such that the center of the crystal is at equal optical distance from D-PBS for clockwise (CW) and counterclockwise (CCW) beams. The working principle for the generation of entangled photons using the current experimental scheme can be found in Refs. [38, 39]. The SPDC photons generated in CW and CCW pump beams have an annular ring intensity profile (as observed using an EMCCD) with mutually orthogonal polarization. The SPDC rings generated by the CW and CCW

pump beams in the Sagnac interferometer are superimposed by the D-PBS, collimated using the lens (L), and then split into two halves by the D-shaped mirrors (DM1, DM2). These halves are subsequently guided by the mirrors (M) toward the polarization analyzers, A1 and A2, comprised of the half-wave plate (HWP), quarter-wave plate (QWP), and polarization beam splitter (PBS). The transmitted photons from each PBS are spectrally filtered using a band-pass filter (BPF) with a full-width at half maximum (FWHM) of ~ 3 nm, centered at 810 nm. The filtered photons are then collected into a single-mode fiber (SMF) via a fiber collimator (FC) and detected using a single-photon counting module (SPCM). Finally, the coincidence detection of photons is performed using a time-to-digital converter (TDC) connected to the SPCMs. Throughout the manuscript, the coincidence window is set to 1.6 ns unless stated otherwise.

4 | Results and Discussion

First, we verified the presence of a geometric phase in the classical pump beam. To achieve this, we set the input beam polarization to GP setup (see Figure 1) as diagonal and used the flip mirror (FM) to direct the beam toward the polarization projection unit (P), which consists of a half-wave plate (H) and a PBS (not shown in Figure 1). By adjusting the angle of the half-wave plate, we set different measurement bases and recorded the power of the horizontally polarized component transmitted through the PBS. The experimental results are presented in Figure 2a.

As evident from Figure 2a, the power of the input beam for a diagonal projection varies sinusoidally from a minimum of 0.004 mW to a maximum of 1.63 mW with the geometric phase, which arises from the orientation of the HWP angle, θ_H in the GP setup. In contrast, for a horizontal/vertical basis, the output

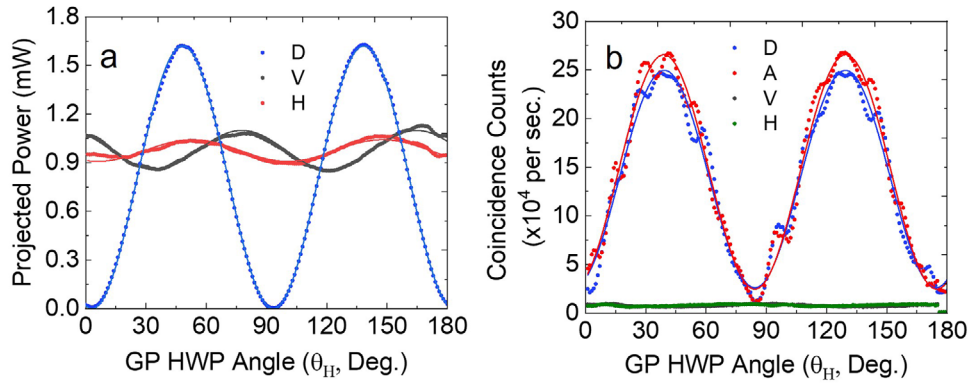


FIGURE 2 | (a) Pump power measurement with GP HWP rotation for polarizer set at 0° , 45° , and 90° for H (red), D (blue), and V (black) basis respectively; (b) Coincidence Counts measurement with GP HWP rotation for H-basis (green), V-Basis (black), D-Basis (blue), A-Basis (red) respectively.

power remains constant at half of the total input power. This observation aligns with theoretical expectations, where the PBS output for diagonal/anti-diagonal and horizontal/vertical bases is given by $P_{D/A} = \sin^2 \phi$ and $P_{H/V} = 1/2$, respectively. These results confirm that the geometric phase is imprinted on the input classical pump beam simply by rotating the half-wave plate in the GP setup. The small oscillations observed in the output power on the horizontal/vertical measurement basis can be attributed to the unoptimized retardance of the waveplates and the related compensation through waveplate tilt used in the experiment.

Further, we studied the effect of the geometric phase of the input pump on the pair photons. In doing so, we removed the flip mirror (FM) from the pump beam path and detected the pair photons of diametrically opposite points of the SPDC ring for different polarization projections using the different settings of the analyzers, A1 and A2. We set analyzer A1 at the polarization settings of H, V, D, and A, and the corresponding orthogonal polarization setting in analyzer A2 for minimum coincidence counts, and rotated the HWP of the GP setup with the results shown in Figure 2b. As evident from Figure 2b, the coincidence counts of the pair photons show the effect of the geometric phase on the D/A basis in agreement with Equation (5), whereas there is no effect in the H/V basis, exactly the same as the pump beam (see Figure 2a). This observation not only confirms the direct transfer of the geometric phase from the classical pump beam to the single photons but also demonstrates a novel approach to measuring the geometric phase of a classical pump beam. Analyzing the coincidence counts of the generated pair photons provides a simple and effective alternative to conventional interferometric setups typically used for phase measurement. Notably, this technique is inherently free from dynamic phase contributions, making it a robust and direct approach for dynamic phase-free geometric phase measurement in quantum optics.

To confirm the transfer of the geometric phase of the classical pump beam to the entangled pair photons, we investigated its effect on the generated quantum state. Keeping the pump power constant at approximately ~ 2 mW, we studied the quantum interference of the generated state from the Sagnac interferometer-based entangled photon source [38, 39] (see Figure 1). Coincidence counts between the entangled photons were recorded while fixing the angle Θ_s of the HWP in analyzer, A1, to project onto various polarization bases, and simultaneously rotating the angle

Θ_i of the HWP in analyzer, A2, from 0° to 180° , for different values of geometric phase ϕ induced by the HWP angle θ_H of the GP setup. The results are presented in Figure 3.

As shown in Figure 3a, for $\phi = 2\theta_H = 0^\circ$, the coincidence counts at four polarization bases, H (black), V (red), D (green), and A (blue) corresponding to $\Theta_s = 0^\circ, 45^\circ, 22.5^\circ$, and 67.5° , respectively, follow the expected sinusoidal modulation. The interference visibility in the H/V and D/A bases was measured to be greater than 0.94 and 0.91, respectively, without applying any corrections to the experimental data. The corresponding measured CHSH Bell parameter $S = 2.63$ confirms the generation of a high-purity entangled state. We repeated this measurement for geometric phases $\phi = 2\theta_H = \pi/4, \pi/2, 3\pi/4$, and π , with results shown in Figure 3b–e. The interference visibility in the H/V bases remained consistent across all values of geometric phase, with slight variations in maximum coincidence counts attributed to changes in the relative amplitudes of the H and V components of the pump beam (see Figure 2a) due to the residual birefringence of the HWP of the GP setup. In contrast, while the D/A basis interference visibility remains high and comparable to that in H/V for $\phi = 0, \pi/2$, and π , no interference is observed in D/A for $\phi = \pi/4$ and $3\pi/4$. This can be understood from Equation (2), representing the generated entangled state in the experiment. Here, the relative phase between $|HH\rangle$ and $|VV\rangle$, originating from the clockwise and counter-clockwise pump beams in the Sagnac loop is $2\phi = 4\theta_H$. As θ_H varies from 0° to 90° , the state transitions from $|\psi_\phi^-\rangle$ to the orthogonal state $|\psi_\phi^+\rangle$ at $\theta_H = 45^\circ$, and then returns to $|\psi_\phi^-\rangle$. Consequently, interference is preserved in all polarization bases except at $\theta_H = \pi/8$ and $3\pi/8$, where the relative phase $e^{-i4\theta_H}$ renders the two-photon amplitudes distinguishable, thereby suppressing interference in the D and A basis.

To further examine the geometric phase-dependent changes in the quantum state, we measured the CHSH Bell parameter S as a function of geometric phase, with results shown in Figure 3f. The experimentally obtained values of S (black dots) follow a sinusoidal variation, and a theoretical fit (black line) reveals the functional form of $S = a + b \cos(4\theta_H)$, closely resembling Equation (7), with fit parameters $a = \sqrt{2}$ and $b = 1.146 < \sqrt{2}$. The small deviation from the ideal maximum $S = 2\sqrt{2}$ is attributed to experimental imperfections such as multiphoton

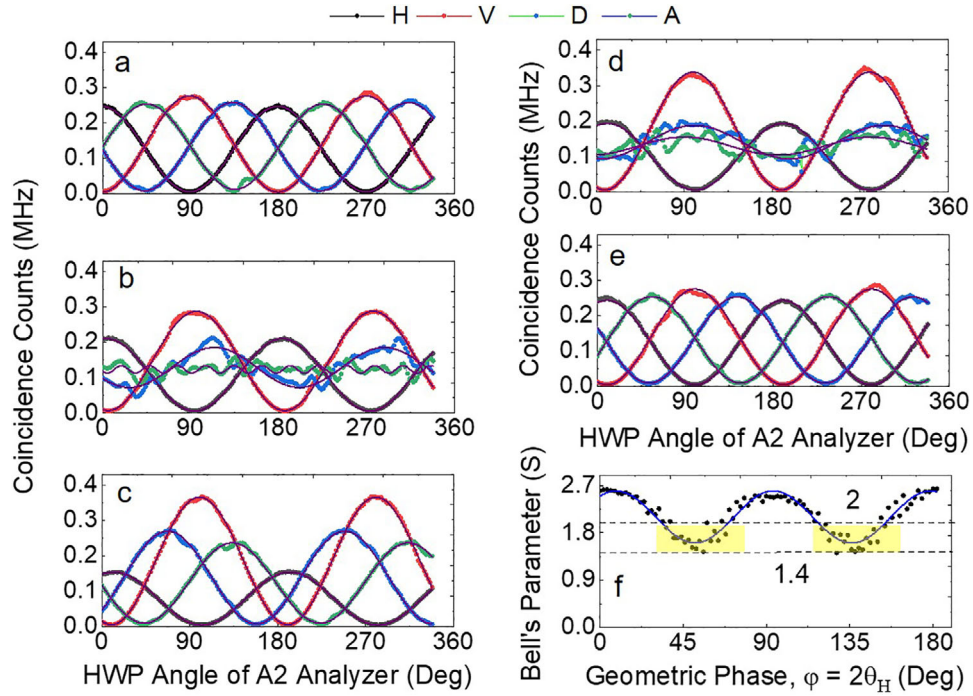


FIGURE 3 | Variation of the coincidence counts as a function of the angle of HWP of analyzer, A2, for the angles 0° (H-Basis, black), 22.5° (D-Basis, green), 45° (V-Basis, red), and 67.5° (A-Basis, blue) of HWP of analyzer, A1, for HWP angle, θ_H , of GP setup at (a) 0°, (b) 22.5°, (c) 45°, (d) 67.5°, (e) 90°. (f) Variation of CHSH Bell's parameter as a function of geometric phase. Yellow shaded regions identify the absence of CHSH violation for the chosen measurement settings. Solid lines are fits to the experimental data points.

contributions, imperfect mode coupling, and optical losses. It is worth noting that none of the presented data have been corrected for background, dark counts, or system losses. Inclusion of such corrections would improve the quantitative agreement of the experimental values with theoretical expectations.

To gain deeper insight into the generated states, we performed quantum state tomography. Based on coincidence counts recorded for different combinations of polarization projections, we reconstructed the real and imaginary parts of the two-qubit density matrix for various geometric phase values, as shown in Figure 4. From the real (top row) and imaginary (bottom row) part of the reconstructed density matrix as shown in Figure 4 it is evident that the quantum state transitions from $|\psi_\phi^-\rangle$ to $|\psi_\phi^+\rangle$ and back to $|\psi_\phi^-\rangle$ for $\theta_H = 0^\circ$, 45° , and 90° , respectively. For intermediate values, $\theta_H = 22.5^\circ$ and 67.5° , the real part of the density matrix remains largely unchanged, while the sign of the off-diagonal elements in the imaginary part flips, indicating a relative phase shift between $|HH\rangle$ and $|VV\rangle$ components. This corresponds to a quantum state of the form $\frac{1}{\sqrt{2}}[|HH\rangle \mp i|VV\rangle]$, in good agreement with the theoretically predicted state in Equation (2).

We further investigated the effect of the controlled geometric phase of the classical pump beam on the generated quantum state by analyzing the entanglement entropy and state fidelity, as shown in Figure 5. Using the Von Neumann entropy of the reduced density matrix approach [45, 46], we calculated the entanglement entropy of the experimentally generated two-qubit state as a function of the HWP angle θ_H in the GP setup. As shown in Figure 5(a), the entanglement entropy obtained

from the experimentally reconstructed density matrices (red dots) remains nearly constant and close to unity across different values of geometric phase ϕ , corresponding to various θ_H values. This agrees well with the theoretical prediction (black dot) calculated from the state given by Equation (2). A small deviation (approximately 10%) in the entanglement entropy around $\theta_H = 45^\circ$ can be attributed to the residual birefringence of the HWP in the GP setup, also evident in Figure 2a, which induces a slight power imbalance between orthogonal polarization components of the classical pump beam, thereby affecting the amplitudes of the generated quantum state. These observations confirm that the generated states are maximally entangled. It is also evident that, although the geometric phase of the classical pump beam influences the type of Bell state produced, it does not affect the degree of entanglement.

On the other hand, the state fidelity of the generated geometric-phase-dependent quantum state, calculated with respect to the Bell state $|\psi_\phi^-\rangle = \frac{1}{\sqrt{2}}(|HH\rangle - |VV\rangle)$, varies from 0.97 to 0.95 as the HWP angle θ_H of the GP setup changes from 0° to 90° , reaching a minimum of zero at $\theta_H = 45^\circ$. The experimental result (dots) is in close agreement with Equation (8). A small offset observed between the experimental and theoretical data can be attributed to the least count error of the rotational stage used to set the angle of the HWP. Such observation confirms the controlled generation and transition between the states $|\psi_\phi^-\rangle$, $|\psi_\phi^+\rangle$, and again $|\psi_\phi^-\rangle$ by simply tuning the geometric phase of the classical pump beam. Moreover, for intermediate values of θ_H , the generated quantum states deviate from ideal Bell states. These intermediate states are basically the maximally entangled Bell-like states, as confirmed by the results of Figure 5a and the standard

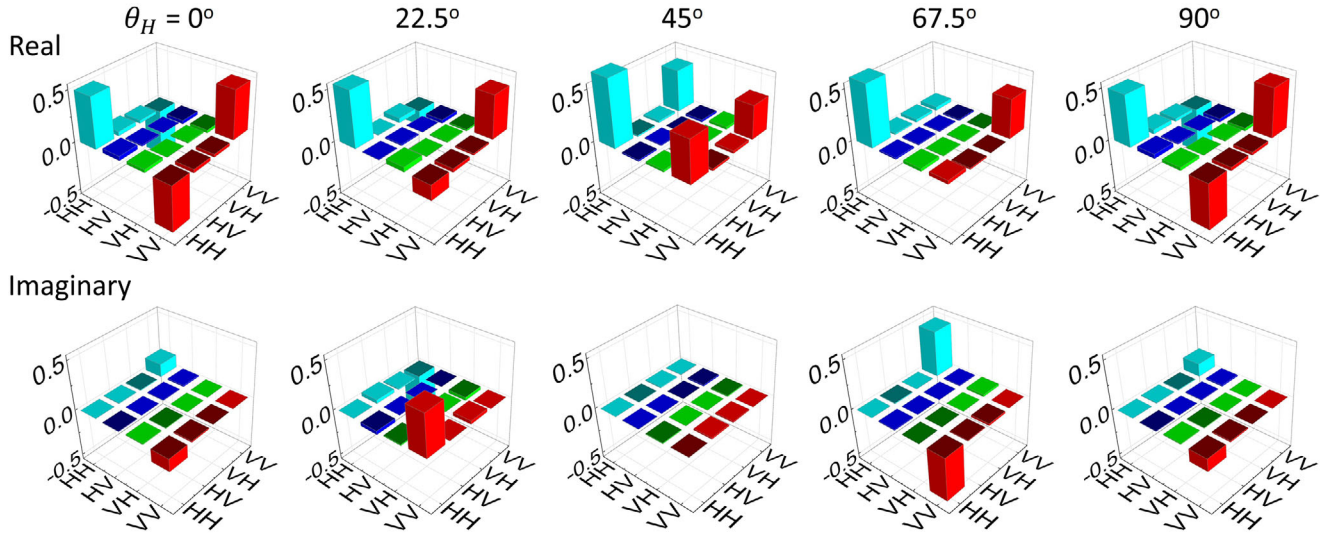


FIGURE 4 | Real (top row) and imaginary (bottom row) part of reconstructed density matrix of generated quantum state for different values of geometric phase, ϕ of the pump beam arising from HWP angle $\theta_H = 0^\circ, 22.5^\circ, 45^\circ, 67.5^\circ$, and 90° .

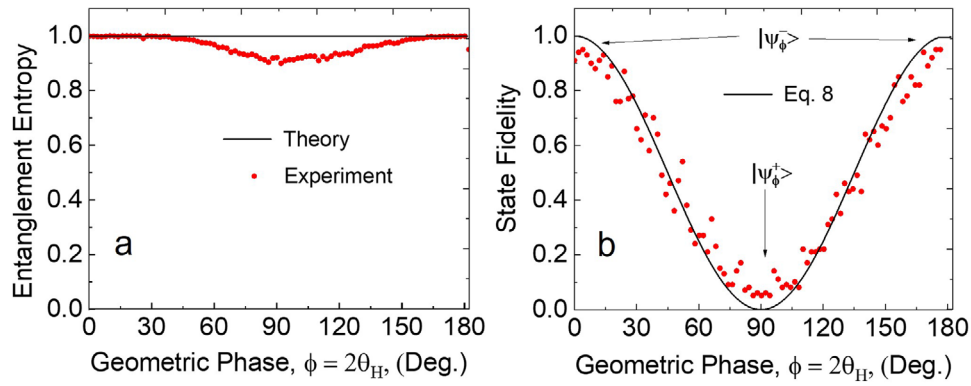


FIGURE 5 | Variation of (a) entanglement entropy and (b) state fidelity of the generated state as a function of the geometric phase of the classical pump beam arising from the HWP angle, θ_H of the GP setup. Dots and solid lines represent experimental and theoretical results, respectively.

density-matrix formalism based quantum state reconstruction presented in Figure 4. Notably, the transformation from $|\psi_\phi^-\rangle$ to $|\psi_\phi^+\rangle$, which is typically realized by applying single-qubit Pauli operators on one qubit of the quantum state, is achieved here through modulation of the geometric phase of the classical pump beam.

5 | Conclusions

In conclusion, we have successfully demonstrated the direct transfer of geometric phase arising from the cyclic evolution of the polarization state of a classical pump beam on the Poincaré sphere to a quantum-entangled state. Using quantum interference measurements, we measured the transferred geometric phase, free from dynamic phase contributions, with high precision. Through quantum state tomography, entanglement entropy, Bell inequality tests, and state fidelity measurements, we confirmed that a simple modulation of the geometric phase of the classical pump beam allows for precise generation, control, and manipulation of the quantum state. This includes a controlled transformation between orthogonal Bell states, while

preserving maximal entanglement fidelity. We also observe the generation of quantum states distinct from Bell states, exhibiting high entanglement entropy, by imprinting different amounts of geometric phase on the pump beam. The current study further highlights the proof-of-principle step toward geometric-phase-controlled quantum operations. The all-optical control method also offers a tunable, stable, compact, and scalable approach for realizing phase-dependent quantum operations, with significant implications for future integrated photonic quantum circuits.

Acknowledgements

The authors acknowledge the support of the Department of Space, Govt. of India. G. K. S. acknowledges the support of the Department of Science and Technology, Govt. of India, through the Technology Development Program (Project DST/TDT/TDP-03/2022).

Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

References

1. S. Pancharatnam, "Generalized Theory of Interference, and Its Applications: Part I. Coherent Pencils," *Proceedings of the Indian Academy of Sciences-Section A* 44 (1956): 247–262.
2. M. Berry, "The Adiabatic Phase and Pancharatnam's Phase for Polarized Light," *Journal of Modern Optics* 34, no. 11 (1987): 1401–1407, <https://doi.org/10.1080/09500348714551321>.
3. P. Bouchal, L. Štrbková, Z. Dostál, R. Chmelík, and Z. Bouchal, "Geometric-Phase Microscopy for Quantitative Phase Imaging of Isotropic, Birefringent and Space-Variant Polarization Samples," *Scientific Reports* 9, no. 1 (2019): 3608.
4. Z. Zhou, S. C. Carrasco, C. Sanner, V. S. Malinovsky, and R. Folman, "Geometric Phase Amplification in a Clock Interferometer for Enhanced Metrology," *Science Advances* 11, no. 18 (2025): eadr6893, <https://doi.org/10.1126/sciadv.adr6893>.
5. C. Kaushik, A. Aadhi, A. Ghosh et al., "Dynamically Tunable Broad-band Output Coupling of Optical Oscillators Based on Non-Cyclic Geometric Phase Mirror," *APL Photonics* 8, no. 11 (2023): 116110.
6. R. Simon, H. J. Kimble, and E. C. G. Sudarshan, "Evolving Geometric Phase and Its Dynamical Manifestation as a Frequency Shift: An Optical Experiment," *Physical Review Letters* 61 (1988): 19–22, <https://doi.org/10.1103/PhysRevLett.61.19>.
7. E. Sjöqvist, "A New Phase in Quantum Computation," *Physics* 1 (2008): 35.
8. R. Simon and N. Mukunda, "Universal SU(2) Gadget for Polarization Optics," *Physics Letters A* 138, no. 9 (1989): 474–480, [https://doi.org/10.1016/0375-9601\(89\)90748-2](https://doi.org/10.1016/0375-9601(89)90748-2).
9. L. Conrads, F. Bontke, A. Mathwieser et al., "Infrared Beam-Shaping on Demand via Tailored Geometric Phase Metasurfaces Employing the Plasmonic Phase-Change Material In₃SbTe₂," *Nature Communications* 16, no. 1 (2025): 3698.
10. B. Décamps, A. Gauguier, J. Vigué, and M. Büchner, "Pancharatnam Phase: A Tool for Atom Optics," *Physical Review A* 96 (2017): 013624, <https://doi.org/10.1103/PhysRevA.96.013624>.
11. C. P. Jisha, S. Nolte, and A. Alberucci, "Geometric Phase in Optics: From Wavefront Manipulation to Waveguiding," *Laser & Photonics Reviews* 15, no. 10 (2021): 2100003, <https://doi.org/10.1002/lpor.202100003>.
12. F. Garoi, I. Nicolae, and P. Prepelita, "Monochromatic Light Measurement via Geometric Phase and Fourier-Transform Spectroscopy Method," *Scientific Reports* 12, no. 1 (2022): 12922.
13. L. A. Alemán-Castaneda, B. Piccirillo, E. Santamato, L. Marrucci, and M. A. Alonso, "Shearing Interferometry via Geometric Phase," *Optica* 6, no. 4 (2019): 396–399, <https://doi.org/10.1364/OPTICA.6.000396>.
14. T. H. Chyba, L. J. Wang, L. Mandel, and R. Simon, "Measurement of the Pancharatnam Phase for a Light Beam," *Optics Letters* 13, no. 7 (1988): 562–564, <https://doi.org/10.1364/OL.13.000562>.
15. M. Ericsson, D. Achilles, J. T. Barreiro, D. Branning, N. A. Peters, and P. G. Kwiat, "Measurement of Geometric Phase for Mixed States Using Single Photon Interferometry," *Physical Review Letters* 94 (2005): 050401, <https://doi.org/10.1103/PhysRevLett.94.050401>.
16. T. Malhotra, R. Gutiérrez-Cuevas, J. Hassett, M. R. Dennis, A. N. Vamivakas, and M. A. Alonso, "Measuring Geometric Phase without Interferometry," *Physical Review Letters* 120 (2018): 233602, <https://doi.org/10.1103/PhysRevLett.120.233602>.
17. O. Arteaga and H. Bendada, "Geometrical Phase Optical Components: Measuring Geometric Phase Without Interferometry," *Crystals* 10, no. 10 (2020): 880.
18. C. Kaushik, A. Ghosh, S. C. Kumar, K. Devi, R. Das, and G. K. Samanta, "Non-Interferometric Measurement of Geometric Phase Through Stimulated Second Harmonic Generation," *Applied Physics Letters* 127, no. 3 (2025): 031101.
19. A. K. Jha, M. Malik, and R. W. Boyd, "Exploring Energy-Time Entanglement Using Geometric Phase," *Physical Review Letters* 101 (2008): 180405, <https://doi.org/10.1103/PhysRevLett.101.180405>.
20. K. Sengupta, S. Dinesh, K. M. Shafi, S. Asokan, and C. Chandrashekar, "Experimental Realization of Universal Quantum Gates and a Six-Qubit Entangled State Using a Photonic Quantum Walk," *Physical Review Applied* 24 (2025): 024012.
21. J. Armstrong, N. Bloembergen, J. Ducuing, and P. S. Pershan, "Interactions Between Light Waves in a Nonlinear Dielectric," *Physical Review* 127, no. 6 (1962): 1918–1939.
22. W.-R. Qi, R. Liu, L.-J. Kong et al., "Pancharatnam–Berry Geometric Phase Memory Based on Spontaneous Parametric Down-Conversion," *Optics Letters* 45, no. 3 (2020): 682–685, <https://doi.org/10.1364/OL.384363>.
23. S.-Y. Huang, H. Jiang, Z.-C. Ren et al., "Observation of High-Order Quantum Pancharatnam–Berry Phase with Structured Photons," *Fundamental Research* (In press) (2024), <https://doi.org/10.1016/j.fmre.2024.11.016>.
24. T. Kim, M. Fiorentino, and F. N. C. Wong, "Phase-Stable Source of Polarization-Entangled Photons Using a Polarization Sagnac Interferometer," *Physical Review A* 73 (Jan 2006): 012316, <https://doi.org/10.1103/PhysRevA.73.012316>.
25. A. Heuer, S. Raabe, and R. Menzel, "Phase Memory Across Two Single-Photon Interferometers Including Wavelength Conversion," *Physical Review A* 90 (2014): 045803, <https://doi.org/10.1103/PhysRevA.90.045803>.
26. Z. Y. Ou, L. J. Wang, X. Y. Zou, and L. Mandel, "Evidence for Phase Memory in Two-Photon Down Conversion Through Entanglement With the Vacuum," *Physical Review A* 41 (1990): 566–568, <https://doi.org/10.1103/PhysRevA.41.566>.
27. V. Kumar, V. Sharma, S. Singh et al., "Imaging Inspired Characterization of Single Photons Carrying Orbital Angular Momentum," *AVS Quantum Science* 4, no. 1 (2022): 015001, <https://doi.org/10.1116/5.0078870>.
28. M. Jabir, N. Apurv Chaitanya, M. Mathew, and G. Samanta, "Direct Transfer of Classical Non-Separable States Into Hybrid Entangled Two Photon States," *Scientific Reports* 7, no. 1 (2017): 7331.
29. D. Zia, N. Dehghan, A. D'Errico, F. Sciarrino, and E. Karimi, "Interferometric Imaging of Amplitude and Phase of Spatial Biphoton States," *Nature Photonics* 17, no. 11 (2023): 1009–1016.
30. A. Einstein, B. Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?," *Physical Review* 47 (1935): 777–780, <https://doi.org/10.1103/PhysRev.47.777>.
31. J. S. Bell, "On the Einstein-Podolsky-Rosen Paradox," *Physica Physique Fizika* 1 (1964): 195–200, <https://doi.org/10.1103/PhysicaPhysiqueFizika.1.195>.
32. A. Aspect, P. Grangier, and G. Roger, "Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedanken Experiment: A New Violation of Bell's Inequalities," *Physical Review Letters* 49 (1982): 91–94, <https://doi.org/10.1103/PhysRevLett.49.91>.
33. S. Mishra, A. Biswas, S. Patil et al., "BBM92 Quantum Key Distribution Over a Free Space Dusty Channel of 200 Meters," *Journal of Optics* 24, no. 7 (2022): 074002, <https://doi.org/10.1088/2040-8986/ac6f0b>.
34. C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, "Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels," *Physical Review Letters* 70 (1993): 1895–1899, <https://doi.org/10.1103/PhysRevLett.70.1895>.
35. S. Barz, "Quantum Computing With Photons: Introduction to the Circuit Model, The One-Way Quantum Computer, and The Fundamental Principles Of Photonic Experiments," *Journal of Physics B: Atomic,*

- Molecular and Optical Physics* 48, no. 8 (2015): 083001, <https://doi.org/10.1088/0953-4075/48/8/083001>.
36. J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, "Proposed Experiment to Test Local Hidden-Variable Theories," *Physical Review Letters* 23 (1969): 880–884, <https://doi.org/10.1103/PhysRevLett.23.880>.
 37. R. C. Jones, "A New Calculus for the Treatment of Optical Systems I. Description and Discussion of the Calculus," *JOSA* 31, no. 7 (1941): 488–493.
 38. M. V. Jabir and G. K. Samanta, "Robust, High Brightness, Degenerate Entangled Photon Source at Room Temperature," *Scientific Reports* 7, no. 1 (2017): 12613, <https://doi.org/10.1038/s41598-017-12709-5>.
 39. S. Singh, V. Kumar, A. Ghosh, A. Forbes, and G. K. Samanta, "A Tolerance-Enhanced Spontaneous Parametric Downconversion Source of Bright Entangled Photons," *Advanced Quantum Technologies* 6, no. 2 (2022): 2200121, <https://doi.org/10.1002/qute.202200121>.
 40. H. Kim, O. Kwon, and H. S. Moon, "Pulsed Sagnac Source of Polarization-Entangled Photon Pairs in Telecommunication Band," *Scientific Reports* 9, no. 1 (2019): 5031.
 41. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, 1995).
 42. M. E. Tunalıoğlu, H. Ö. Çıldıroğlu, and A. U. Yilmazer, "On the Geometric Phases in Entangled States," *Communications Faculty of Sciences University of Ankara Series A2-A3 Physical Sciences and Engineering* 65, no. 2 (2022): 142–151.
 43. H. Cildiroglu, "Testing Bell–CHSH Inequalities Using Topological Aharonov–Casher and He–McKellar–Wilkins Phases," *Annals of Physics* 465 (2024): 169684, <https://doi.org/10.1016/j.aop.2024.169684>.
 44. H. Cildiroglu and A. Yilmazer, "Investigation of the Aharonov–Bohm and Aharonov–Casher Topological Phases for Quantum Entangled States," *Physics Letters A* 420 (2021): 127753, <https://doi.org/10.1016/j.physleta.2021.127753>.
 45. M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2010).
 46. C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, "Mixed-State Entanglement and Quantum Error Correction," *Physical Review A* 54 (1996): 3824–3851, <https://doi.org/10.1103/PhysRevA.54.3824>.